PHYSICAL REVIEW E, VOLUME 65, 056602

# Propagation properties of chirped soliton pulses in optical nonlinear Kerr media

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(Received 9 October 2001; published 15 April 2002)

An investigation is made of the formation of a single soliton, or a pair of solitons, from initially nontransform limited pulses in a nonlinear Kerr medium having anomalous dispersion. A qualitative physical explanation is given for the formation of soliton pairs. Approximate solutions for the amplitudes and the velocities of the generated solitons are established and corroborated by numerical solutions of the Zakharov-Shabat eigenvalue problem.

DOI: 10.1103/PhysRevE.65.056602 PACS number(s): 42.65.Tg

## I. INTRODUCTION

The degradation of the soliton content of initially nontransform limited sech-shaped pulses propagating in a nonlinear Kerr medium having anomalous dispersion has been studied in a number of investigations and for many different applications. The case of a pulse which initially is sech shaped and has a quadratic phase variation in time, i.e., the concomitant frequency chirp is linear, was investigated numerically by Hmurcik and others [1–4]. The results demonstrated that the amplitude of the asymptotically emerging solitons decreased monotonously with increasing chirp strength. At a certain chirp, the soliton character was lost completely and the asymptotic propagation properties were that of a linear pulse. In addition, for initially higher-order solitons, the characteristic soliton number decreased in steps at certain critical chirp strengths. An important observation from the present point of view is the fact that the pulses never split into separating subpulses or soliton pairs. A special situation, that has attracted considerable interest [5,6], is the case when the initial complex pulse amplitude can be written as $A\,\mathrm{sech}^{1+if}(\tau)$. This implies that the pulse is sech shaped, but that the phase, $\phi(\tau)$, varies as $\phi(\tau) = f\ln[\mathrm{sech}(\tau)]$ and the associated chirp frequency is $\omega_c = f\tanh(\tau)$. The chirp frequency is still linear for small $\tau$, but the increase, although still monotonous, saturates at finite values as $\tau\to\pm\infty$. For this particular choice of chirp variation, the corresponding inverse scattering method can be carried out to give explicitly the properties of the asymptotically emerging solitons. The result is qualitatively the same as in the previous case, the soliton content is degraded, in the sense that the soliton number decreases in steps, the emerging soliton pulse has an amplitude that decreases for increasing chirp strength and eventually, above a certain chirp threshold, only linear dispersive radiation exist asymptotically.

In stark contrast to this smooth degradation of the soliton content with increasing chirp strength stands another type of pulse dynamics [7,8]. At a certain critical chirp strength, the smooth degradation is broken by a discontinuous transition into a qualitatively different behavior where the initial pulse splits into two or more separating soliton pairs. A number of analytical investigations has also been made to explain the features observed in the numerical simulations. An early effort was made by Lewis [9], where the semiclassical limit of the Zakharov-Shabat scattering problem was analyzed and later further important information was obtained by Kaup, El-Reedy, and Malomed [10]. Yet, both these works considered simplifying limits of, e.g., rectangular initial amplitudes and/or piecewise constant chirp variations. Several investigations [11–15] have relied on direct variational methods, which have given good approximate solutions for the degradation of the soliton content with increasing chirp strength, however, primarily for linear chirp variations. An interesting approach based on the invariants of the nonlinear Schrödinger equation has also been presented [16], for obtaining information on the asymptotically emerging soliton/soliton pair. Nevertheless, it is fair to say that in spite of many important results, no clear physical understanding has emerged regarding what properties of the initial amplitude and phase that determine the character of the asymptotically appearing soliton pulses.

The present paper investigates in some detail the problem of the soliton content of an initially nontransform limited sech-shaped pulse. The quantitative analysis is restricted to the case when the initial pulse, in the absence of phase variations, corresponds to an $N=2$ soliton. The result indicates that the properties of asymptotically emerging solitons (if any) depend crucially on the magnitude, but also on the form, of the initial chirp. In particular, the splitting of an initial pulse into separating soliton pulse pairs only occurs for certain classes of initial chirp functions. The analysis is based on a combination of qualitative reasoning, approximate analysis using the invariants of the nonlinear Schrödinger equation, and numerical solutions of the Zakharov-Shabat eigenvalue problem.

## II. QUALITATIVE ANALYSIS

We consider the nonlinear evolution of optical pulses as determined by the normalized form of the nonlinear Schrö-

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dinger equation (NLS) in the case of anomalous dispersion, i.e.,

$$i\frac{\partial\Psi}{\partial x}+\frac{1}{2}\frac{\partial^2\Psi}{\partial\tau^2}+|\Psi|^2\Psi=0 \tag{1}$$

with initial conditions which are sech shaped, but nontransform limited, i.e., the initial pulse has a nontrivial phase variation, $\phi(\tau)$,

$$\Psi(0,\tau)=A_0\,\mathrm{sech}(\tau)e^{if\phi(\tau)}. \tag{2}$$

The properties of the corresponding initial chirp frequency, $\omega_c=-f\,d\phi/d\tau$, play a decisive role for the subsequent pulse evolution. A qualitative understanding of the physical implications of the frequency chirping can be obtained as follows: The variation of chirp frequency with $\tau$ implies that different parts of the pulse are given different local pulse velocities, which consequently tend to disperse and broaden the pulse. However, counteracting and competing with this evolution is the nonlinear effect which for anomalous dispersion tends to compress and keep the pulse together. The outcome of this competition depends crucially on the properties of the chirp variation. The following qualitatively different forms of phase variations will be considered in the present paper,

$$\phi(\tau)=\tau^2, \tag{3a}$$

$$\phi(\tau)=\ln[\mathrm{sech}(\tau/T)], \tag{3b}$$

$$\phi(\tau)=\mathrm{sech}(\tau/T). \tag{3c}$$

For a phase of the form (3a), the local chirp frequency changes linearly and it can be expected that at some point in the pulse, the chirp will be strong enough to overcome the nonlinear attraction and to break loose part of the pulse as dispersive radiation. For a small chirp parameter $f$ this occurs only far out in the wings of the pulse and little energy will be lost in the form of dispersive radiation. However, for increasing chirp strength, $f$, more and more energy will be lost and the asymptotic soliton will contain less and less energy. Finally for a sufficiently large $f$, the soliton content is completely destroyed and the total pulse disperses asymptotically as a linear pulse.

In case (3c) the chirp is $\omega_c=f\,\mathrm{sech}(\tau/T)\tanh(\tau/T)/T$. This variation is qualitatively different from case (3a) in the sense that the chirp-induced velocities have extrema around which there tend to build up subpulses which move faster or slower than the central part. Counteracting this separation are: linear dispersion which tends to broaden the pulses and make them merge into a single broad pulse and nonlinear effects which tend to form single pulses. The latter effect works in the same direction as the linear dispersion when the subpulses are strongly overlapping, but will oppose the dispersive broadening if the subpulses are sufficiently separated. These considerations imply that if the initial chirp is strong enough, the pulse can be expected to be torn apart into two separating subpulses, and if these pulses are energetic enough, nonlinear effects will transform them into symmetrically moving and separating solitons. It is obvious that the width $T$ of the phase variation, which determines the location of the local extrema of the chirp variation, also plays an important role for the possibility of splitting the pulse into soliton pairs. Clearly for large $T$ the extrema of the chirp are moved far out into the wings of the pulse and soliton pair creation should become more difficult.

The qualitative picture above suggests that soliton pair creation requires a nonmonotonous chirp. Yet this is not true. Consider case (3b) where the chirp $\omega_c=f\tanh(\tau/T)/T$ is monotonous, but bounded. Even if the chirping rate here saturates, it seems possible that the combination of linear dispersion and nonlinear attraction may still manage to tear off parts of the original pulse and to create separating subpulses in the form of solitons. On the other hand, this requires a more delicate interplay between the chirping and the nonlinearity as compared to the previous cases. It is interesting to note that for $T=1$, the initial condition corresponds to $\Psi(0,\tau)=A\,\mathrm{sech}^{1+if}(\tau)$ for which the soliton content can be obtained exactly in closed form [5,6]. The solution predicts only regular soliton degradation i.e., for increasing chirp strength, the soliton amplitude, and the soliton number decrease, but no separating soliton pairs appear.

Thus, in view of the above qualitative reasoning, it may be conjectured that in case (3b), if the rise time of the chirp variation is long enough as compared with the pulse width ($T>T_{\mathrm{crit}}\simeq 1$), only regular radiative soliton degradation will occur, whereas soliton pair creation should occur for short rise times ($T<T_{\mathrm{crit}}\simeq 1$) and sufficiently strong chirping. This conjecture is confirmed by the numerical simulations made in the present paper for the cases $T=2$ and $T=1/2$, respectively, cf. Fig. 1.

On the other hand, even if this explains the features observed for pulses with moderate amplitudes, it cannot be the complete qualitative picture. In situations of high amplitudes, the modulational instability will tend to create many soliton embryos on the "broad" pedestal pulse. If, e.g., the chirping is linearly varying it will tend to assign a specific (but different) mean velocity to each soliton embryo. If the nonlinearity is strong enough, it can be expected to create true solitons of the soliton embryos. Since the velocities of the created solitons will increase with distance from the initial pulse center, the result should be a sequence of soliton pairs moving out in a fan-shaped pattern in space time, in good qualitative agreement with the numerical simulation results presented by others [7,8]. A more quantitative analysis of the chirp-induced soliton degradation and splitting will be presented in the next section.

## III. QUANTITATIVE ANALYSIS

An approximate analytical approach for determining the properties of the solitons emerging from initially chirped sech-shaped pulse was presented by Maĭmistov [16]. The main idea of this approach is that if an initial $N=2$ soliton, i.e., $\Psi(0,\tau)=2\,\mathrm{sech}(\tau)$ is transformed, by initial chirping, into two solitons or a soliton pair, the corresponding amplitudes $(A_1,A_2)$ and velocities $(V_1,V_2)$, constitute a set of four unknown quantities. These soliton parameters can be

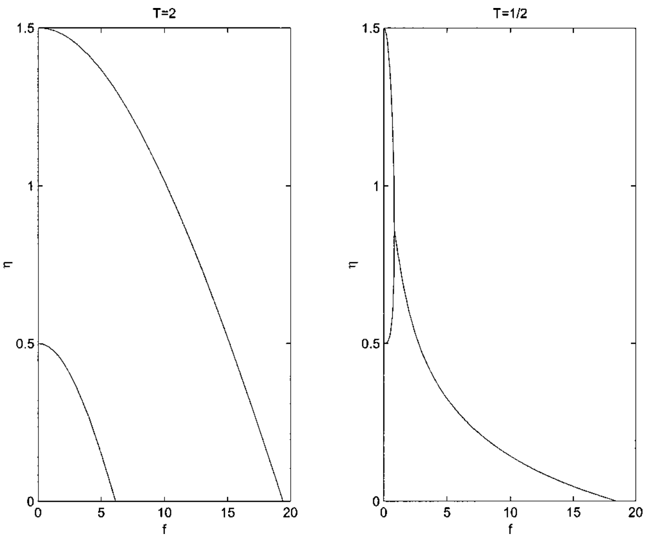


FIG. 1. Degradation of the $N=2$ soliton for the initial phase condition given by Eq. (3b) and $T=2$ and $T=1/2$ respectively; numerical solution of the Zakharov-Shabat eigenvalue problem.

determined by using the first four invariants of the NLS equation, which read

$$I_1=\int_{-\infty}^{\infty}|\Psi|^2 d\tau,$$

$$I_2=\int_{-\infty}^{\infty}\left(\Psi^*\frac{\partial\Psi}{\partial\tau}-\Psi\frac{\partial\Psi^*}{\partial\tau}\right)d\tau,$$

$$I_3=\int_{-\infty}^{\infty}\left(\left|\frac{\partial\Psi}{\partial\tau}\right|^2-|\Psi|^4\right)d\tau, \tag{4}$$

$$I_4=\int_{-\infty}^{\infty}\left(\Psi\frac{\partial^3\Psi^*}{\partial\tau^3}+3\Psi|\Psi|^2\frac{\partial\Psi^*}{\partial\tau}\right)d\tau.$$

Four independent equations can be obtained by evaluating the invariants for the initial condition and for the asymptotic solution consisting of two fundamental solitons, if we neglect possible overlap effects between the solitons. The chirp will not affect the energy of the pulse and hence $I_1=8$ for all chirp strengths $f$. The even invariants vanish, $I_2=I_4=0$, since the initial condition is even. However, the third invariant depends on the chirp strength and is given by,

$$I_3=-\frac{56}{3}+\left(8\int_0^{\infty}\operatorname{sech}^2(\tau)[\phi'(\tau)]^2 d\tau\right)f^2. \tag{5}$$

The most general form of the fundamental soliton is

$$\Psi=A\operatorname{sech}[A(\tau-\tau_0-Vx)]\exp\left[i(V\tau-(V^2-A^2)\frac{x}{2}+\phi_0)\right] \tag{6}$$

and the corresponding values of the invariants are

$$\begin{aligned} I_1&=2A,\\ I_2&=4iAV,\\ I_3&=2AV^2-\frac{2}{3}A^3,\\ I_4&=-2iAV(A^2-V^2). \end{aligned} \tag{7}$$

Depending on whether the chirp strength $f$ is smaller or larger than a critical value $f_c$, either a two-soliton breather or two symmetrically diverging solitons, respectively, are obtained as solutions of the algebraic system. The solutions are [16]

$$\begin{aligned} &V_1=V_2=0, && A_1=A_2=2,\\ &A_1=2+\sqrt{1-(f/f_c)^2}, && V_1=+\sqrt{(f/f_c)^2-1},\\ &A_2=2-\sqrt{1-(f/f_c)^2}, && V_2=-\sqrt{(f/f_c)^2-1}, \end{aligned} \tag{8}$$

where

$$f_c=\frac{1}{\sqrt{\int_0^{\infty}\operatorname{sech}^2(\tau)(\phi'(\tau))^2 d\tau}} \tag{9}$$

is the critical chirp strength for which the solution changes character. Expressed in eigenvalues, the result can be written in the compact form,

$$\zeta=\xi+i\eta=i\left(1\pm\frac{1}{2}\sqrt{1-(f/f_c)^2}\right). \tag{10}$$

In the case of vanishing chirp, $f=0$, Eq. (10) yields the exact eigenvalues $\zeta=i3/2$ and $\zeta=i/2$, which is due to the fact that the initial condition 2 sech($\tau$) is reflectionless and no dispersive radiation is present.

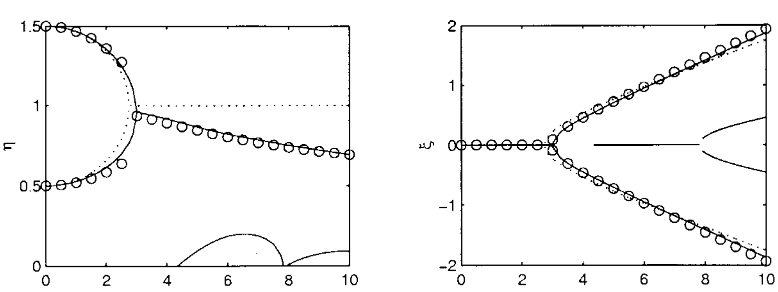


FIG. 2. Soliton degradation for the initial condition given by Eq. (13). Solid line, numerical solution of the Zakharov-Shabat eigenvalue problem. Dashed line, approximate solution [Eq. (10)], Circles, approximate solution [Eq. (10) with corrections according to Eqs. (21) and (22)]. Note the appearance of additional discrete eigenvalues at $f\approx4.5$ and at $f\approx8.0$.

Inherent in this approximation procedure is the assumption that the solution can be accurately described by two solitons and in particular that no (or at least little) energy is lost as dispersive radiation. Although this scheme gives good approximations in several important situations [16], it may also give rise to totally misleading results if the underlying assumption is poorly fulfilled. In the paper by Maĭmistov and Sklyarov [16], a critical chirp was found to exist for the case of quadratic phase variation. This result predicts that pulse splitting, with the appearance of two separating solitons, should occur for large enough chirp strengths, a nonphysical result for the $N=2$ soliton case [3,4]. Furthermore, if we calculate the critical chirp strength for case (3b) in Eq. (3), we find a finite value of $f_c$ for all values of $T$. This implies that, independent of the value of $T$, it should be possible to split the initial pulse into a separating soliton pair, provided the chirp is strong enough. However, this is not true. From our previous qualitative discussion we expect that pulse splitting should occur only when the chirp width, $T$, is smaller than unity and indeed for $T=1/2$ we obtain $f_c=1/\sqrt{2(\pi-2)}\approx 0.66$, which is in qualitative agreement with the numerical results shown in Fig. 1. However, for $T=1$, we obtain $f_c=\sqrt{3}\approx 1.73$, whereas the exact analytical solution for the Zakharov-Shabat problem [5,6] asserts that no pulse splitting should occur. Similarly for $T=2$, we find that the critical chirp strength increases to $f_c=2/\sqrt{\pi-3}\approx 5.32$, but according to Fig. 1, no pulse splitting occurs, as expected from the qualitative discussion. The physical reason for this shortcoming of the analytical approach given in Ref. [16] is that dispersive radiation and/or additional soliton generation plays an important role in the asymptotic pulse evolution.

A further assumption made in Ref. [16] was that the invariants corresponding to an $N$-soliton solution, $\psi_s(x,\tau)$, could be rewritten as the sum of the invariants corresponding to each individual soliton, $\psi_s^j(x,\tau)$, i.e.,

$$I_n(\psi_s(x,\tau))=\sum_{j=1}^{N} I_n(\psi_s^j(x,\tau)). \tag{11}$$

Although this is evidently a legitimate approach in the case of separating solitons, there were some doubts raised in Ref. [16] about applying the same procedure to the case of solitons with zero velocity, i.e. nonseparating (and overlapping) solitons. However, the approach can indeed be put on a firm basis by noticing that the soliton contributions to the invariants can be expressed directly in terms of the eigenvalues, $\zeta_k$, of the scattering problem, as follows, cf. Ref. [17]:

$$\begin{aligned}
I_1&=2i\sum_{k=1}^{N}\zeta_k^*-\zeta_k=4\sum_{k=1}^{N}\eta_k,\\
I_2&=4\sum_{k=1}^{N}(\zeta_k^2)^*-\zeta_k^2=-16i\sum_{k=1}^{N}\xi_k\eta_k,\\
I_3&=\frac{8i}{3}\sum_{k=1}^{N}(\zeta_k^3)^*-\zeta_k^3=16\sum_{k=1}^{N}\xi_k^2\eta_k-\frac{1}{3}\eta_k^3,\\
I_4&=4\sum_{k=1}^{N}(\zeta_k^4)^*-\zeta_k^4=-32i\sum_{k=1}^{N}\xi_k\eta_k(\xi_k^2-\eta_k^2).
\end{aligned} \tag{12}$$

This has two advantages, first the calculations leading to Eq. (7) are simplified, second and more important, it is clear that the resulting equations are equally valid for the case when the asymptotic solution is a breather soliton; no overlap effects between solitons occur.

We will demonstrate an extension of this approach which accounts for dispersive and/or additional soliton generation. In order to be explicit we will consider the two-soliton initial condition

$$\Psi(0,\tau)=2\,\mathrm{sech}(\tau)\exp[if\,\mathrm{sech}(\tau)]. \tag{13}$$

The critical chirp strength obtained for this case is $f_c=\sqrt{15/2}\approx 2.74$, which is in good agreement with the critical chirp strength found by a full numerical solution of the Zakharov-Shabat scattering equations. However, it is also clear from the numerical results that energy is lost from the solitons to dispersive radiation and, in some parameter regimes, also to additional solitons cf. Fig. 2. In particular, for increasing chirp strength the discrepancy between numerical results and theoretical predictions increases.

In the present paper, we will generalize the approximate approach by including also the fifth invariant, which provides additional information about the pulse dynamics and makes it possible to account for energy loss due to dispersive waves and/or the presence of additionally generated solitons. Hence, we assume that the asymptotic solution consists of two main solitons and a small additional pulse. We linearize the invariants $I_3$ and $I_5$ in order to find the corrections to the soliton parameters given in Eq. (8). In this linearization procedure the contributions to $I_3$ and $I_5$ coming from the small additional pulse vanish and to lowest order, it only contributes to the energy invariant, $I_1$. The subsequent results imply a significant improvement on the results obtained previously [16].

The fifth invariant is

$$I_5=\int_{-\infty}^{\infty}\left(\left|\frac{\partial^2\Psi}{\partial\tau^2}\right|^2+2|\Psi|^6-\left(\frac{\partial|\Psi|^2}{\partial\tau}\right)^2-6\left|\frac{\partial\Psi}{\partial\tau}\right|^2|\Psi|^2\right)d\tau, \tag{14}$$

and its value, calculated for the initial condition is

$$I_5=\frac{488}{5}-\frac{1072}{105}f^2+\frac{64}{315}f^4. \tag{15}$$

The value again depends on the amount of chirping. The contribution to the invariant from the soliton part of the solution is given by

$$I_5 = \frac{32i}{5}\sum_{k=1}^{N} (\zeta_k^5)^* - \zeta_k^5$$
$$= 64\sum_{k=1}^{N} \eta_k(\eta_k^2 - \xi_k^2)^2 - \frac{4}{5}\eta_k^5. \quad (16)$$

Assuming the total solution to consist of two diverging solitons with common amplitude and sign reversed velocities and an additional pulse, which for simplicity will be taken as a stationary and soliton-shaped pulse, with eigenvalue $\zeta = i\mu$, we obtain

$$I_3(\eta,\xi,\mu) = 16\left(2\xi^2\eta - \frac{2}{3}\eta^3 - \frac{1}{3}\mu^3\right),$$
$$I_5(\eta,\xi,\mu) = 64\left(2\eta(\eta^2-\xi^2)^2 - \frac{8}{5}\eta^5 + \frac{1}{5}\mu^5\right). \quad (17)$$

The corrections $\delta\eta$ and $\delta\xi$ are found from the linearized equations

$$I_k^0 = I_k(\eta_0,\xi_0,0) + \frac{\partial I_k}{\partial \eta}(\eta_0,\xi_0,0)\,\delta\eta$$
$$+ \frac{\partial I_k}{\partial \xi}(\eta_0,\xi_0,0)\,\delta\xi + \frac{\partial I_k}{\partial \mu}(\eta_0,\xi_0,0)\,\delta\mu, \quad (18)$$

were $I_k^0$ denotes the value of the invariant corresponding to the initial condition, and $(\eta_0,\xi_0)$ denotes the previous solution given in Eq. (10). From the third invariant we directly obtain a connection between the corrections in the real and imaginary parts of the eigenvalue, viz.

$$\delta\xi = \frac{\eta_0^2 - \xi_0^2}{2\xi_0\eta_0}\,\delta\eta. \quad (19)$$

The fifth invariant makes the system inhomogeneous and determines the value of the correction in $\eta$,

$$\delta\eta = -\frac{I_5^0 - I_5(\eta_0,\xi_0,0)}{128(\eta_0^2+\xi_0^2)^2}. \quad (20)$$

Simplifying the results we obtain for the case $f > f_c$,

$$\delta\eta = -\frac{3}{7}\frac{(f/f_c)^2[1+(f/f_c)^2]}{[3+(f/f_c)^2]^2},$$
$$\delta\xi = \frac{5-(f/f_c)^2}{4\sqrt{(f/f_c)^2-1}}\,\delta\eta. \quad (21)$$

In the complementary case $(f < f_c)$ when the asymptotic soliton corresponds to a breather solution we obtain

$$\delta\eta_1 = \frac{3}{28}\frac{(f/f_c)^2[1+(f/f_c)^2]}{(2+\sqrt{1-(f/f_c)^2})^2\sqrt{1-(f/f_c)^2}},$$
$$\delta\eta_2 = -\frac{3}{28}\frac{(f/f_c)^2[1+(f/f_c)^2]}{(2-\sqrt{1-(f/f_c)^2})^2\sqrt{1-(f/f_c)^2}}. \quad (22)$$

We emphasize that the properties of the additional pulse only enters explicitly in the first energy invariant from which we, by linearization, obtain that the energy, $E$, of the additional pulse is given by $E = -8\delta\eta$. This implies that our analysis covers both the case when the additional pulse corresponds to a stationary third soliton and the case of dispersive radiation.

The qualitative speculations and the quantitative predictions have been compared with the eigenvalues obtained by a full numerical solution of the Zakharov-Shabat scattering problem. The agreement is found to be very good, except in the close vicinity of $f_c$ where the solution changes character and for large $f$ where the solution is dominated by dispersive radiation and the linearization procedure is violated, cf. Fig. 2. In connection with Fig. 2 we emphasize that the numerical results show an unusual feature: The conventional picture of the effect of chirping on nonlinear pulse evolution is that increasing chirp strength tends to decrease the number of eigenvalues and eventually leads to a purely continuous spectrum. However, in the case considered in Fig. 2, the opposite occurs in the sense that at certain chirp strengths eigenvalues are pulled out of the continuum (one eigenvalue at $f \approx 4.5$ and two at $f \approx 8.0$). Thus, e.g., at $f = 9$, the eigenvalue problem has four discrete different eigenvalues.

## IV. SUMMARY

The soliton content in an initially chirped pulse, propagating in a nonlinear Kerr medium, can be found by solving the corresponding Zakharov-Shabat scattering problem. Albeit linear, this scattering problem can be solved in closed form for very few cases and usually resort must be taken to approximate or numerical techniques. In this paper we first discuss qualitatively the effect of an initial chirp of different forms on the subsequent development of the pulse. This discussion gives important physical insight into the expected pulse dynamics. A detailed quantitative investigation is then made of the influence of an initial chirp on a two-soliton breather. It is found that, for certain phase functions and chirp strengths, the pulse is split into separating solitons. However, another scenario is also possible where degradation of the two soliton breather and formation of dispersive radiation is observed. An approximate solution scheme based on the first four invariants of the nonlinear Schrödinger equation was presented by Maĭmistov and Sklyarov [16]. In this method it is assumed that the asymptotic solution consists of only two solitons. The quality of the approximate solution depends crucially on the validity of this assumption and in some parameter regimes the accuracy of the solution rapidly deteriorates. In order to improve the approximate solution we have included the possibility of a small pulse appearing asymptotically in addition to the two solitons. Since this step increases the number of unknowns, we have also included the next order invariant $I_5$ in order to obtain a consistent set of equations. To lowest order in energy, the inclusion of the additional pulse will only affect the first invariant expressing the energy conservation. However, this will change the previous values of the soliton parameters, which now must be corrected as compared to the soliton parameters obtained from the four first invariants. In order to calculate the corresponding corrections to the speed and amplitudes of the solitons we linearize the invariants $I_3$ and $I_5$ around the previous solution. We have studied in detail the case when the initial pulse is given by $\Psi(0,\tau)=2\,\mathrm{sech}(\tau)\exp[if\,\mathrm{sech}(\tau)]$ and it was found that the approximate solution is in very good agreement with a full numerical solution of the Zakharov-Shabat scattering equations for a large range of parameters.


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